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The critical behaviour of directed Lévy flight on fractal lattices

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Abstract. Lévy flight is a random walk in which the step length r is a random variable with probability distribution proportional to $1/r^{1+u}$, where u is the Lévy parameter and $0<u<\infty$. In this paper we study the critical behaviour of fully directed Lévy flight on Sierpinski carpets using the Monte Carlo method. The obtained critical exponents $\nu_{\parallel}$ is independent of the parameter u, but $\nu_{\perp}$ is found to be dependent on u. This seems to be interesting compared with the directed Lévy flight on ordinary Euclidean lattices previously discussed by Hu and one of the authors, for which $\nu_{\parallel}$ and $\nu_{\perp}$ are independent of u. These results indicate that directed Lévy flights on different fractals belong to different universality classes.

Recently there has been an increasing interest in the problems of various random walks on fractal lattices and on percolation clusters such as random walk on Sierpinski carpets [1], self-avoiding random walks on diluted networks [2], long-range random walks on percolation clusters [3], random walks on multifractal lattices [4] and the problems of Lévy flight on Euclidean and fractal lattices [5].

Lévy flight is similar to random walk, except that the steps are not necessarily to next neighbours [7]. Instead, the probability $p(r)$ for a step to have a step length r is proportional to $1/r^{1+u}$, with $u>0$. These walks are called Lévy flights since $p(r)$ is a distribution of Lévy type. The properties of such a random walk are strikingly different from those of ordinary random walks. Recently, some variants of the original Lévy flight such as node-avoiding Lévy flight (NALF) and path-avoiding Lévy flight (PALF) have been proposed to study an extensive range of physical phenomena, such as chaos and turbulence, adsorption of polymer chains, and pattern recognition. Yao and Hu [5] studied the extension of node-avoiding Lévy flight to include a preferred direction in the path of the walk, which is called 'directed Lévy flight'. Their simulation results indicated that the critical behaviour of directed Lévy flight on ordinary Euclidean lattices is independent of u. In this paper, we study the critical behaviour of such a directed Lévy flight on fractal lattices.

At first, we construct a fractal lattice, a Sierpinski carpet, with the fractal dimension of 1.892 (see figure 1) which is a kind of regular fractal. Using the fractal-cell generation method we can form an infinite fractal lattice with the structure of the Sierpinski carpet (see figure 2). The introduction of preferred direction in such systems gives rise to two independent correlation lengths $R_{\parallel}$ and $R_{\perp}$, parallel and perpendicular to the preferred

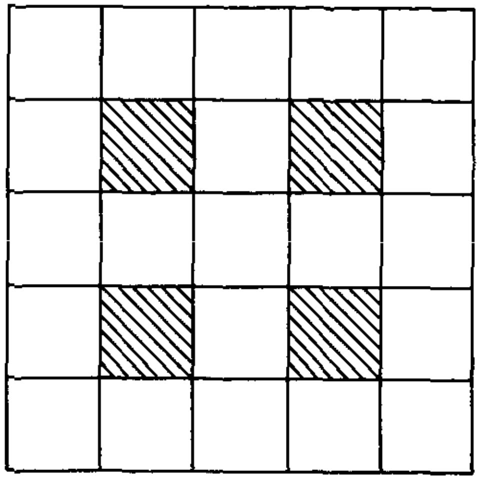
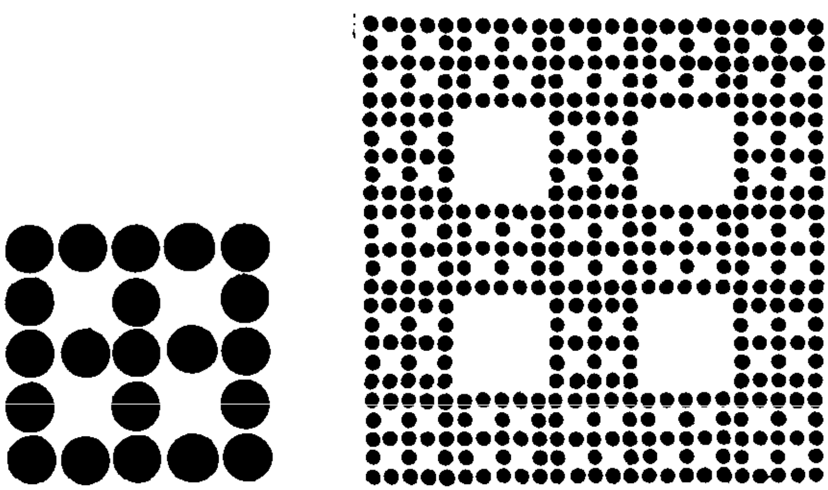


Figure 1. The Sierpinski Carpet with $b=5$, $l=2$, $d_f=1.892$ treated in this work. The hatched area indicates the hole that was cut off.



(*a*) (*b*)

Figure 2. The fractal-cell generation method for a fractal lattice with the Sierpinski carpet structure as in figure 1. (*a*) The first-stage cell, containing 21 ($b^2-l^2=21$) lattice sites. (*b*) The second-stage cell, containing 21 first-stage cells.

direction, respectively. In the case of directed SAW the corresponding exponents $\nu_\parallel$ and $\nu_\perp$ have been obtained with the values $\nu_\parallel=1.0$ and $\nu_\perp=0.67\pm0.02$ for the fractal dimension of $d_f=1.892$ [6]. For directed SAW $\nu_\perp$ is fractal dimension dependent, for instance $\nu_\perp=0.59\pm0.01$ for $d_f=1.975$ and $\nu_\perp=0.83\pm0.03$ for $d_f=1.792$ [6].

In order to study the critical behaviour of directed Lévy flight on fractal lattices we have performed a Monte Carlo simulation in our Honeywell DPS8 machine. The probability of making a step with step length r was chosen as $[r^{-u}-(r+1)^{-u}]$. This is because the probability for a step to have a length greater than r is assumed to decrease as r. The maximum step length was chosen as $r=6$.

The simulation results indicate that the scaling behaviour of $\overline{R_\parallel^2}$ (or $\overline{R_\perp^2}$) and N is quite good. We express $\overline{R_\parallel^2}$ and $\overline{R_\perp^2}$ for $N \to \infty$ as

$$\overline{R_\parallel^2} \sim N^{2\nu_\parallel} \qquad \overline{R_\perp^2} \sim N^{2\nu_\perp}$$

noting that $\overline{R_\parallel^2}$ and $\overline{R_\perp^2}$ imply an averaging over a large number of different origins.

Figure 3 shows the plots of $\frac{1}{2}\log_2 \overline{R_\parallel^2}$ and $\frac{1}{2}\log_2 \overline{R_\perp^2}$ against $\log_2 N$. From the slopes of the straight-line fits we obtained the critical exponents for directed Lévy flight on the fractal lattice of $d_f = 1.892$.

u	$\nu_\parallel$	$\nu_\perp$
0.5	0.96 ± 0.01	0.54 ± 0.02
1.0	0.96 ± 0.01	0.57 ± 0.02
1.5	0.97 ± 0.01	0.59 ± 0.01
2.0	0.97 ± 0.01	0.62 ± 0.01
2.5	0.97 ± 0.01	0.63 ± 0.02

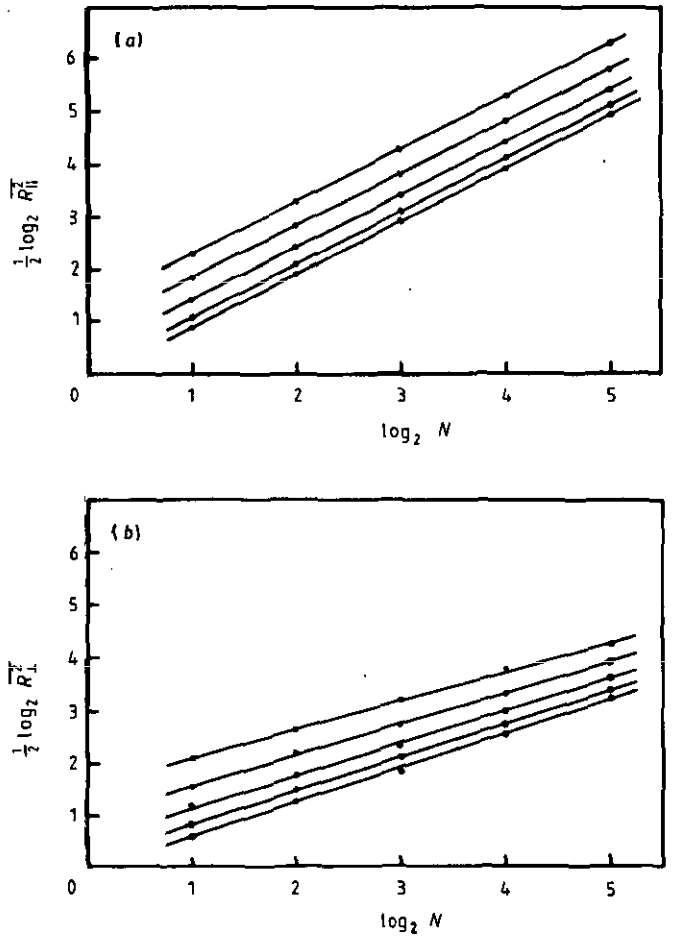


Figure 3. (a) Plot of $\frac{1}{2}\log_2 \overline{R_\parallel^2}$ against $\log_2 N$ for, from top to bottom, $u = 0.5$, 1.0, 1.5, 2.0, 2.5. (b) A plot of $\frac{1}{2}\log_2 \overline{R_\perp^2}$ against $\log_2 N$ for, from top to bottom, $u = 0.5$, 1.0, 1.5, 2.0, 2.5.

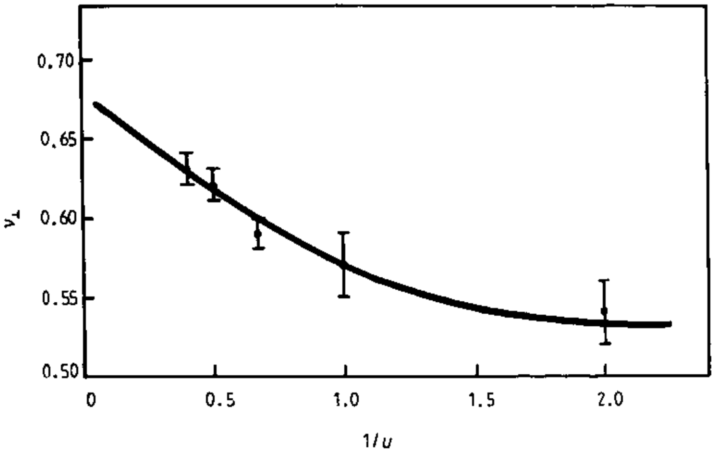


Figure 4. Dependence of $\nu_\perp$ on the parameter u.

The dependence of $\nu_\perp$ on the parameter u is shown in figure 4. It is interesting to notice that when $u \to \infty$ $(1/u \to 0)$ the critical exponent $\nu_\perp$ for directed Lévy flight approaches 0.67, which is the same value that we obtained for directed SAW on the Sierpinski carpet in [6]. This is reasonable since, when $u \to \infty$, the walker will only take one step forward or to the right, which means the Lévy flight for $u \to \infty$ becomes actually the directed SAW. On the other hand when $u \to 0$ $(1/u \to \infty)$, the critical exponent $\nu_\perp$ for directed Lévy flight approaches $\frac{1}{2}$, which is in agreement with the value for an ordinary random walk on Euclidean space. Since when $u \to 0$, according to the expression for probability $[r^{-u} - (r+1)^{-u}]$, only the largest step length dominates and the probability for other step lengths can be neglected. Thus the walker will always take the largest step length in the case of $u \to 0$. The walker can fly across the voids easily for any kind of fractal structure, so in principle the fractal structure has no influence on the flight. So, in the case of $u \to 0$ the directed Lévy flight naturally is equivalent to ordinary directed SAW on Euclidean lattices.

From these results we know that $\nu_\parallel$ for directed Lévy flight on a fractal lattice is independent of the parameter u, but $\nu_\perp$ is found to be dependent on u. This is interesting compared with the directed Lévy flight on Euclidean lattices previously discussed by Yao and Hu, where $\nu_\parallel$ and $\nu_\perp$ are both independent of u [5]. These results indicate that directed Lévy flights on a fractal lattice and on an Euclidean lattice belong to different universality classes.

Acknowledgments

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